Space-time Beamforming with Knowledge-Aided Constraints

Adaptive Sensor Array Processing Workshop
March 11-12, 2003
MIT Lincoln Laboratory, Lexington, MA

Jameson Bergin
ISL, Inc.
8130 Boone Blvd.
Suite 500
Vienna, VA 22182
jsb@isl-inc.com

Christopher M. Teixeira
ISL, Inc.
8130 Boone Blvd.
Suite 500
Vienna, VA 22182
cmt@isl-inc.com

Paul M. Techau
ISL, Inc.
8130 Boone Blvd.
Suite 500
Vienna, VA 22182
pmt@isl-inc.com

Joseph R. Guerci
DARPA
Special Projects Office
3701 N. Fairfax Dr.
Arlington, VA 22203
jguerci@darpa.mil

INFORMATION
SYSTEMS
LABORATORIES, INC.

SPO
Special Projects Office

Outline

- Background
- Knowledge-aided Constraints
- Performance Results
- Summary

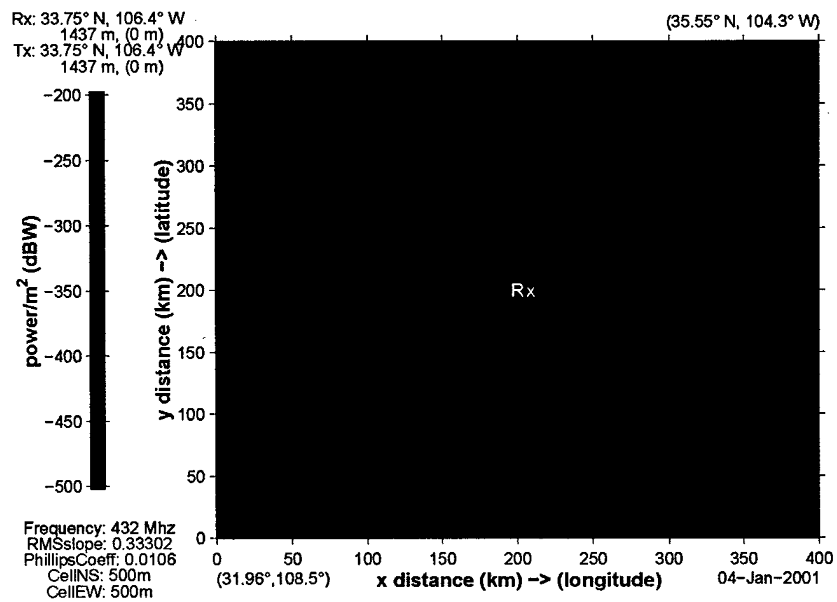
Background

- **Real-world radar clutter environments depend on site-specific factors including:**
  - Terrain
  - Ground cover type
- **Site-specific clutter modeling is fundamental to understanding STAP performance in real-world settings**
- **This has led to the development of site-specific performance bound techniques**
  - Thermal noise limited performance is optimistic for systems operating in real-world environments
  - Theory is based on ideal site-specific clutter covariance modeling
- **It is logical that the models used in site-specific performance analyses could also be used when processing the radar data to potentially improve radar performance**
- **This is a major goal of DARPA's Knowledge-Aided Sensor Signal Processing and Expert Reasoning (KASSPER) Program**

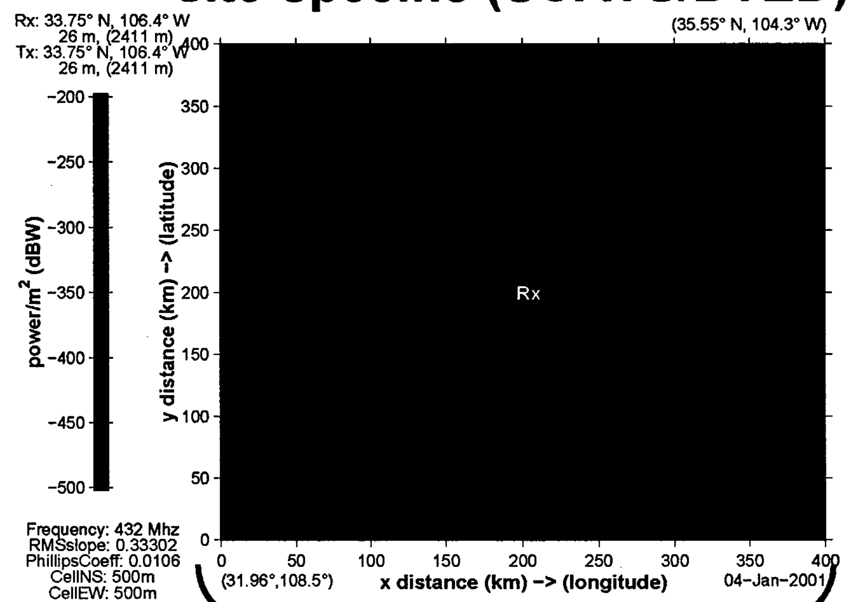
[redacted]

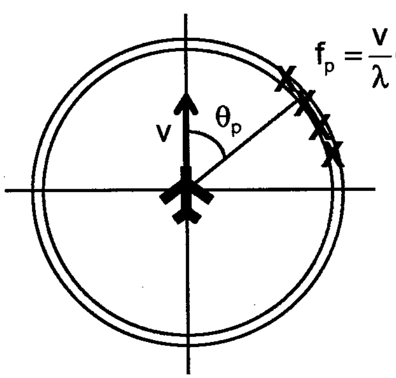
Site-specific Clutter Modeling

bald earth



site-specific (SCATS/DTED)





$$f_p = \frac{v}{\lambda}\cos(\theta_p)$$

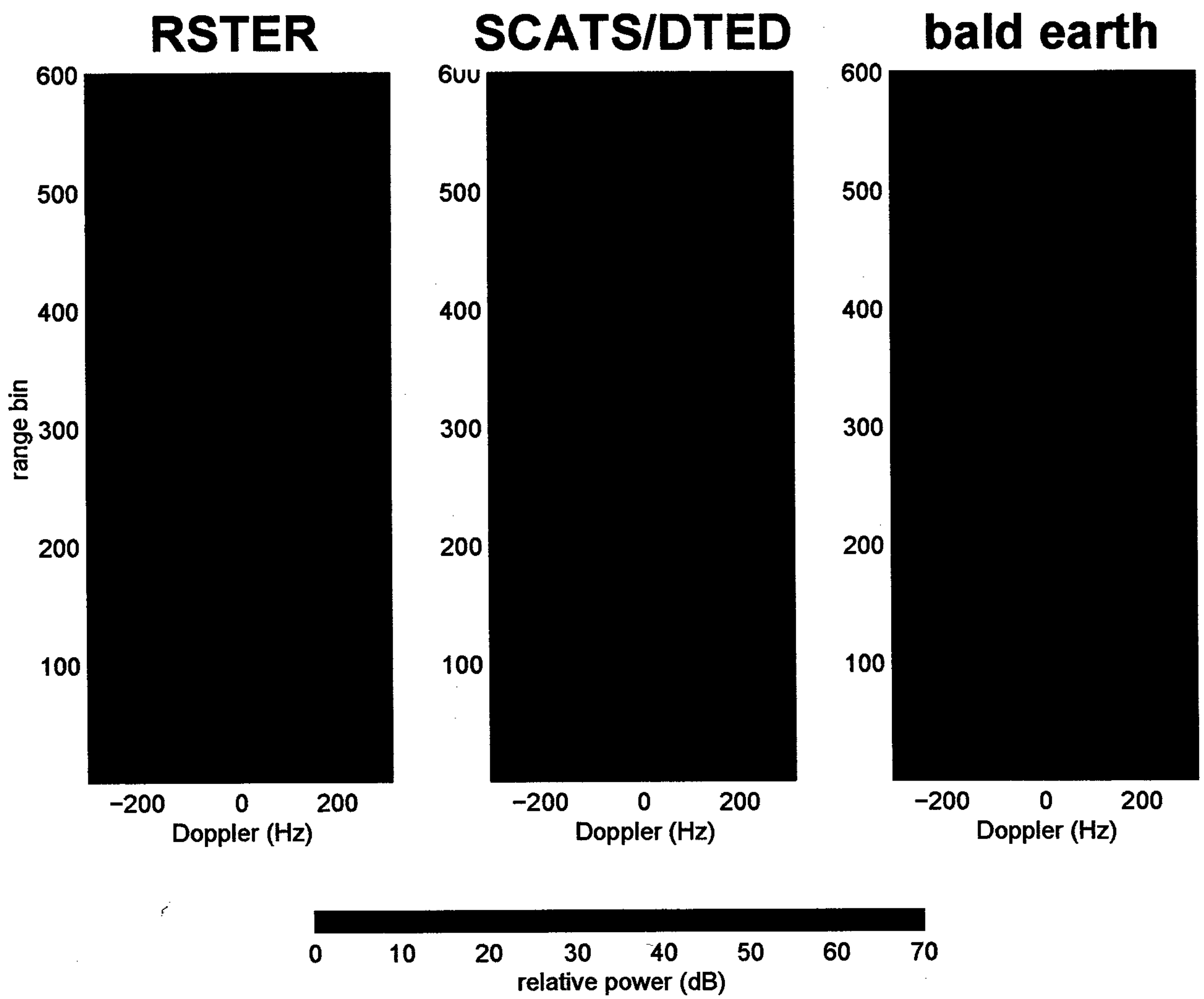
ICM
calibration errors
channel mismatch

$$\mathbf{R}_c = \sum_{p=1}^{P_c} |\alpha_p|^2 \mathbf{v}(\theta_p, f_p)\mathbf{v}^H(\theta_p, f_p) \circ \mathbf{T}_p$$

sum over all scattering patches in a given range bin

[redacted]

Mountain Top Monostatic Clutter



- Range-Doppler clutter maps shown for RSTER and simulations
- Simulation results shown both with and without DTED
- Simulation w/ DTED results in a significantly better match to the experimental data
- Site-specific models capture a majority of the clutter features

Knowledge-Aided Signal Processing

- **The *a priori* knowledge will typically be used in two ways**
  - Indirect: exploit knowledge sources to segment training data, etc.
  - Direct: exploit knowledge sources to place nulls in the beamformer pattern

notch width varies little with range

significant notch width variation with range

- **This presentation develops a methodology for using *a priori* knowledge *directly* in the space-time beamforming solution**
- **Clutter cancellation based on *a priori* knowledge alone will typically not result in adequate performance**
- **Focus will be on techniques that combine or "blend" adaptive and deterministic filtering**
- **The performance of these filtering techniques will be a function of how well the system is calibrated**

[redacted]

Outline

- Background
- Knowledge-aided Constraints
- Performance Results
- Summary

Interference Modeling

- **Assume the clutter signal plus thermal noise model**

$$\mathbf{x} = \mathbf{x}_c \circ \mathbf{t} + \mathbf{n}$$

$\circ$ - Hadamard product (element-wise product)

- **The modulation will typically be small**

$$\mathbf{t} = \mathbf{1} + \mathbf{d}$$

$\mathbf{d}$ **is zero-mean, variance << 1**

- **Clutter signal with small modulation**

$$\mathbf{x} = \mathbf{x}_c + \mathbf{x}_c \circ \mathbf{d} + \mathbf{n}$$

- **Clutter correlation matrix**

$$E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{R}_{xx} = E\{\mathbf{x}_c\mathbf{x}_c^H\} + E\{\mathbf{x}_c\mathbf{x}_c^H\} \circ E\{\mathbf{d}\mathbf{d}^H\} + \sigma^2\mathbf{I}$$

$$= \mathbf{R}_c + \underbrace{\mathbf{R}_c \circ \mathbf{T} + \sigma^2\mathbf{I}}_{\text{unknown component}}$$

"known" component

[redacted]

Knowledge-Aided Quadratic Constraints

- **The usual optimization problem:**

$$\min_{\mathbf{w}} E\{|\mathbf{w}^H\mathbf{x}|^2\} \quad \text{s.t.} \quad \mathbf{w}^H\mathbf{v} = 1 \quad \rightarrow \quad \mathbf{w} = \frac{\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{v}}{\mathbf{v}^H\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{v}}$$

- **Incorporate covariance model as a quadratic constraint**

$$\min_{\mathbf{w}} E\{|\mathbf{w}^H\mathbf{x}|^2\} \quad \text{s.t.} \quad \begin{aligned} \mathbf{w}^H\mathbf{v} &= 1 \\ \mathbf{w}^H\mathbf{R}_c\mathbf{w} &= 0 \\ \mathbf{w}^H\mathbf{w} &= \delta \end{aligned}$$

want weights to be orthogonal to *a priori* clutter model

this is the KA part

- **Gives:**

$$\mathbf{w} = \frac{(\mathbf{R}_{\mathbf{xx}} + \beta_d\mathbf{R}_c + \beta_L\mathbf{I})^{-1}\mathbf{v}}{\mathbf{v}^H(\mathbf{R}_{\mathbf{xx}} + \beta_d\mathbf{R}_c + \beta_L\mathbf{I})^{-1}\mathbf{v}} = \frac{(\mathbf{R}_{\mathbf{xx}} + \mathbf{Q})^{-1}\mathbf{v}}{\mathbf{v}^H(\mathbf{R}_{\mathbf{xx}} + \mathbf{Q})^{-1}\mathbf{v}}$$

"colored loading"

[redacted]

Pre-Filter Interpretation

- Colored loading beamformer can be expressed as:

$$\mathbf{w} = \frac{\mathbf{Q}^{-1/2}(\mathbf{Q}^{-1/2}\mathbf{R}_{xx}\mathbf{Q}^{-1/2}+\mathbf{I})^{-1}\mathbf{Q}^{-1/2}\mathbf{v}}{\mathbf{v}^H\mathbf{Q}^{-1/2}(\mathbf{Q}^{-1/2}\mathbf{R}_{xx}\mathbf{Q}^{-1/2}+\mathbf{I})^{-1}\mathbf{Q}^{-1/2}\mathbf{v}}$$

- Notice that,

$$\mathbf{w}^H\mathbf{x} = \tilde{\mathbf{w}}^H\tilde{\mathbf{x}}$$

it will generally be easier to estimate the covariance of the pre-filtered data than the original data because it is likely to have a lower *effective* rank

- Where,

$$\tilde{\mathbf{x}} = \mathbf{Q}^{-1/2}\mathbf{x}$$ pre-filtered or whitened data

$$\tilde{\mathbf{w}} = \frac{(\mathbf{R}_{\tilde{x}\tilde{x}}+\mathbf{I})^{-1}\tilde{\mathbf{v}}}{\tilde{\mathbf{v}}^H(\mathbf{R}_{\tilde{x}\tilde{x}}+\mathbf{I})^{-1}\tilde{\mathbf{v}}}$$ optimal weights for whitened data

$$\tilde{\mathbf{v}} = \mathbf{Q}^{-1/2}\mathbf{v}$$ whitened constraint

[redacted]

Knowledge-Aided Linear Constraints

- **Re-write quadratic constraint using the eigen-decomposition of the *a priori* clutter model, $R_c = U^H D U$ (dominant subspace)**

$$\mathbf{w}^H \mathbf{R}_c \mathbf{w} = 0 \Rightarrow \mathbf{w}^H(\mathbf{U}\mathbf{D}\mathbf{U}^H)\mathbf{w} = 0 \Rightarrow (\mathbf{w}^H\mathbf{U})\mathbf{D}(\mathbf{U}^H\mathbf{w}) = 0$$

$$\Rightarrow \mathbf{w}^H\mathbf{U} = \mathbf{0} \quad \because \mathbf{D}$$ **has strictly positive diagonal elements (also has dimensions << R_c e.g., Brennan's Rule)**

- **A set of linear constraints**

$$\min_{\mathbf{w}} \; E\{|\mathbf{w}^H\mathbf{x}|^2\} \quad \text{s.t.} \quad \mathbf{w}^H\mathbf{v} = 1, \quad \mathbf{w}^H\mathbf{U} = \mathbf{0}, \quad \mathbf{w}^H\mathbf{w} = \delta$$

desire weights to be orthogonal to *a priori* clutter model

this is the KA part

- **Gives:**

$$\mathbf{w} = \frac{\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{U})^{-1}\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1})\mathbf{v}}{\mathbf{v}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{U})^{-1}\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1})\mathbf{v}} = \frac{\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{P}\mathbf{v}}{\mathbf{v}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{P}\mathbf{v}}$$

$$\overline{\mathbf{R}}_{\mathbf{xx}} = \mathbf{R}_{\mathbf{xx}} + \beta_L \mathbf{I}$$

[redacted]

Quadratic vs. Linear Constraints

- The two solutions:

$$\mathbf{w} = \frac{(\overline{\mathbf{R}}_{xx} + \beta_d \mathbf{R}_c)^{-1}\mathbf{v}}{\mathbf{v}^H(\overline{\mathbf{R}}_{xx} + \beta_d \mathbf{R}_c)^{-1}\mathbf{v}} \qquad \mathbf{w} = \frac{\overline{\mathbf{R}}_{xx}^{-1}\mathbf{P}\mathbf{v}}{\mathbf{v}^H\overline{\mathbf{R}}_{xx}^{-1}\mathbf{P}\mathbf{v}}$$

- Manipulation of the quadratic constraint solution using the matrix inversion lemma and eigen-decomposition highlights the difference between the two solutions

$$\overline{\mathbf{R}}_{xx}^{-1}\mathbf{P} = \overline{\mathbf{R}}_{xx}^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{U}^H\overline{\mathbf{R}}_{xx}^{-1}\mathbf{U})^{-1}\mathbf{U}^H\overline{\mathbf{R}}_{xx}^{-1})$$

$$(\overline{\mathbf{R}}_{xx} + \beta_d \mathbf{R}_c)^{-1} = \overline{\mathbf{R}}_{xx}^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{U}^H\overline{\mathbf{R}}_{xx}^{-1}\mathbf{U} + \boxed{\frac{1}{\beta_d}\mathbf{D}^{-1}})^{-1}\mathbf{U}^H\overline{\mathbf{R}}_{xx}^{-1})$$

- The two solutions are equivalent in the limit of infinite loading and/or large clutter model eigenvalues
- Since the linear constraints cause the weights to be precisely orthogonal to the known covariance the quadratic constraint achieves this constraint only approximately

Advantages of Quadratic Constraints

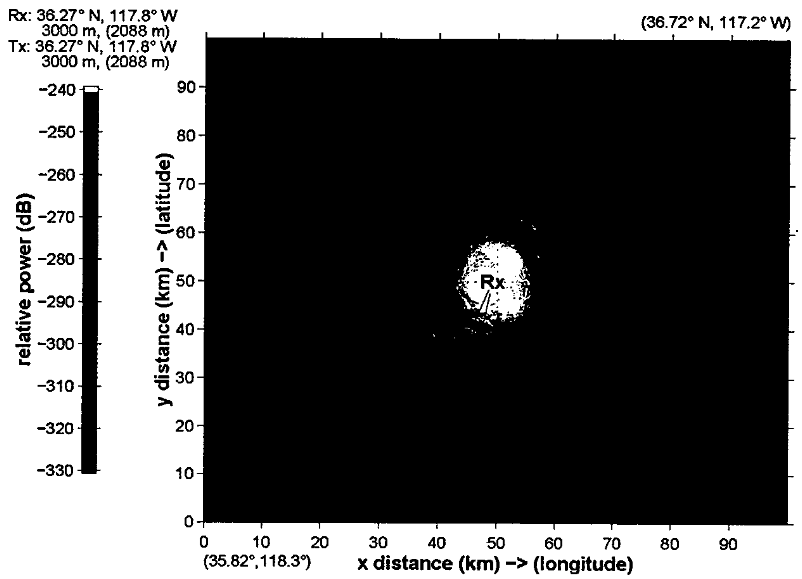
- Quadratic constraints can be implemented more efficiently in both the covariance and the data domain

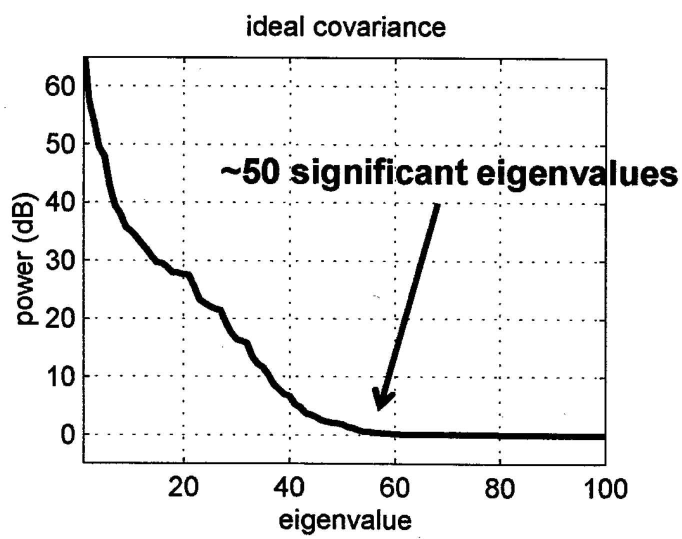
- Quadratic constraints offer a "blending" mechanism between the adaptive and deterministic beamformers

Outline

- Background
- Knowledge-aided Constraints
- Performance Results
- Summary

KASSPER Simulated Data Cube





Parmeter	Value
RF frequency	1240 MHz
Bandwidth	10 MHz
PRF	1984 Hz
Peak Power	15 kW
Duty factor	10%
Noise figure	5 dB
System losses	9 dB
Platform speed	100 m/s
Platform altitude	3 km
Transmit aperture	8 vertical x 11 horizontal
Receive aperture*	8 vertical x 1 horizontal
Horizontal antenna spacing	10.9 cm
Vertical antenna spacing	14.07 cm
Number of receive sub-apertures	11
Front-to-back ratio	25 dB

- **Site-specific data set generated under KASSPER program**
- **Heterogeneous clutter, ground vehicles, ICM, calibration errors**
- **We will focus on the problem of detecting slow moving targets in heterogeneous clutter → work with clutter-only data**

[redacted]

Colored Loading Matrix

- **Assume a ring of scatterers every 0.2° around the platform at the desired range bin**
  - No knowledge about terrain included
  - No knowledge about calibration errors (~5°-10° phase errors)
  - No knowledge about ICM included
  - No knowledge about backlobe level or Tx pattern included
  - Only platform heading, speed, and PRF are assumed known
- **Compute a matrix that represents the ground clutter (subspace):**

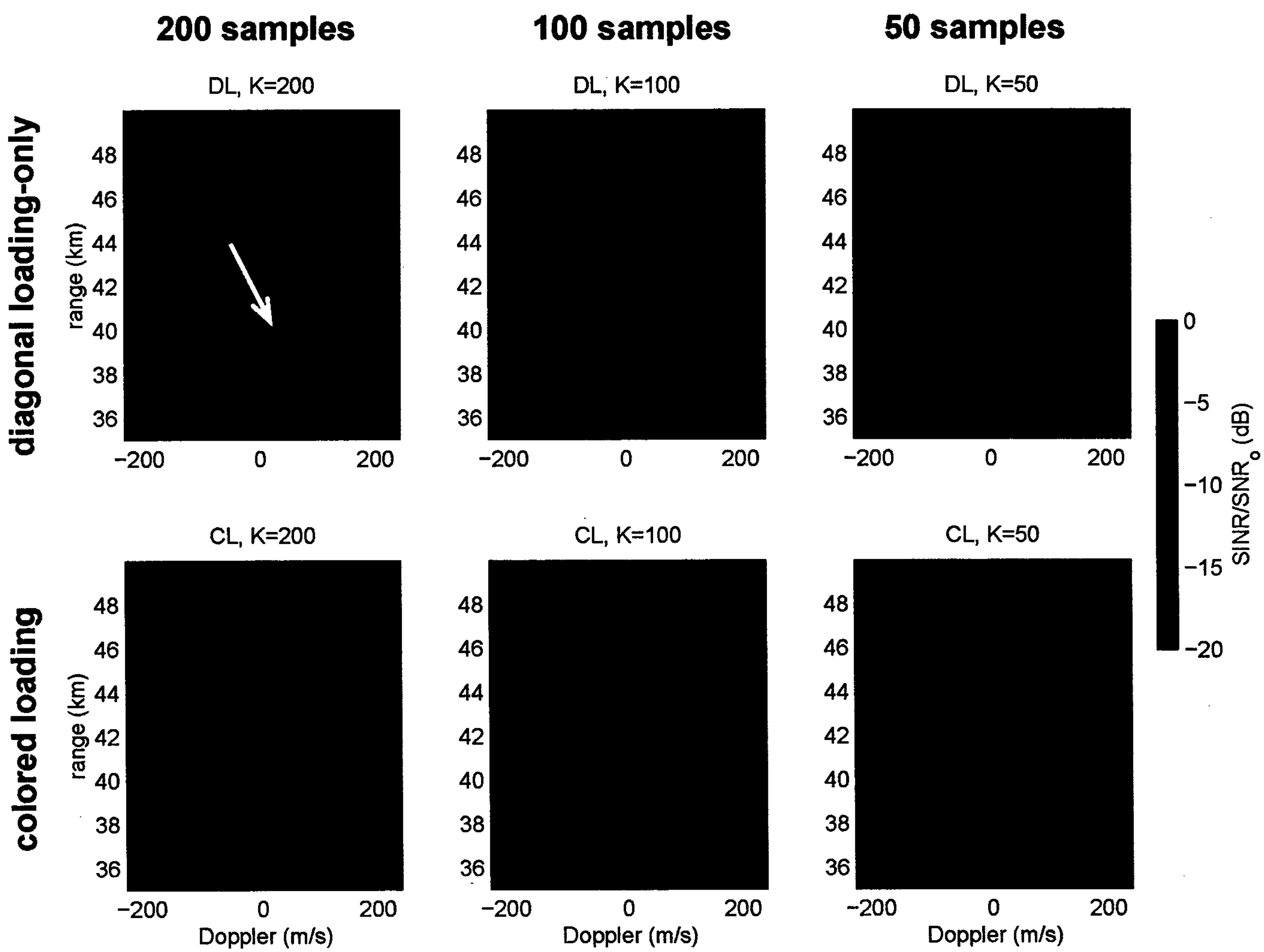
$$\mathbf{R}_c = \sum_{p=1}^{N_c} \mathbf{v}(\theta_p, f_p)\mathbf{v}(\theta_p, f_p)^H$$

- **Scale this matrix and add to the diagonally-loaded sample covariance matrix:**

$$\mathbf{w} = \kappa(\mathbf{R}_s + \beta_L \mathbf{I} + \beta_d \mathbf{R}_c)^{-1}\mathbf{s}$$

- **Note: there are more efficient methods for computing this form of $\mathbf{R}_c$**

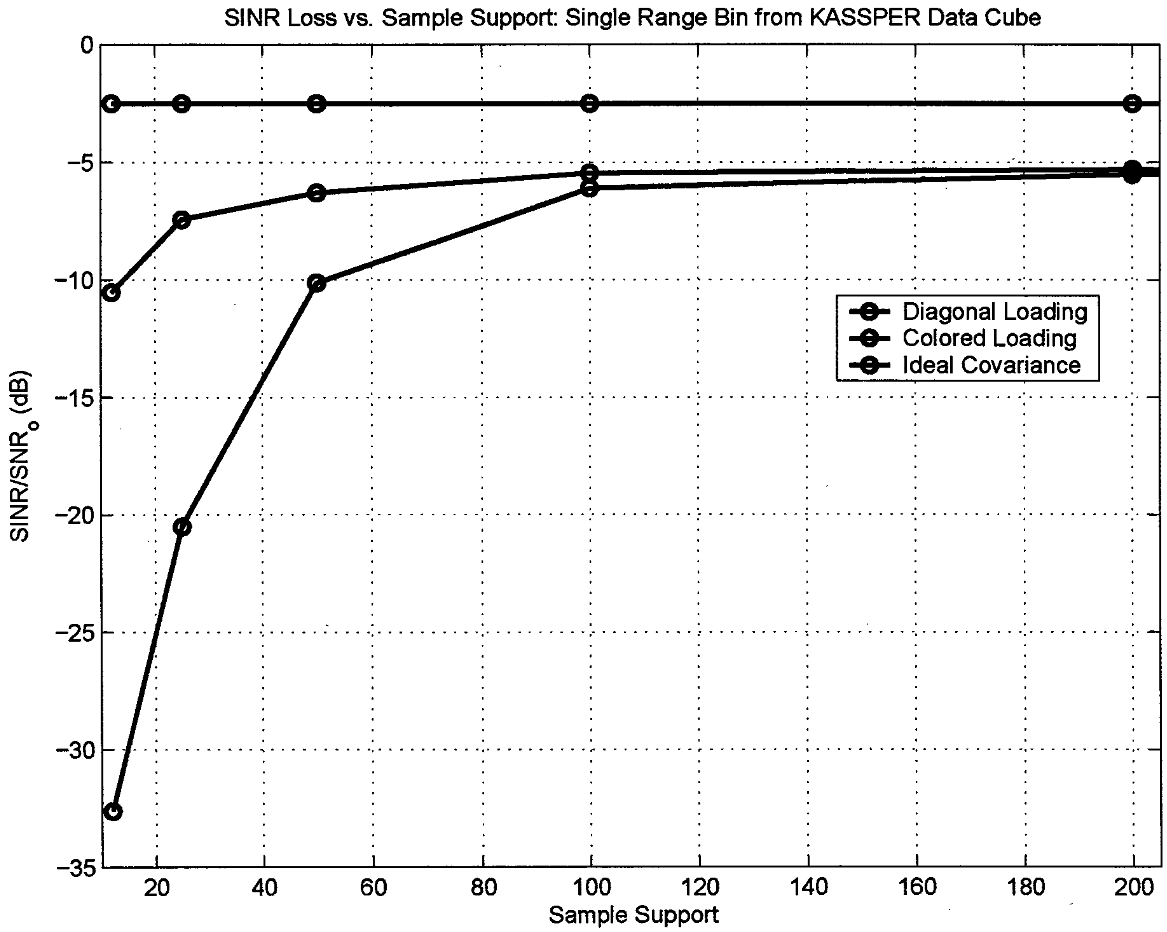
SINR Loss Surfaces



- **Loading levels:**
  β_L = 0 dB
  β_d = 30 dB
- **Colored loading beamformer is more robust to reductions in sample support**
- **Full-DoF SMI**
  - 32 pulses
  - 11 elements

[redacted]

Beamformer Convergence Summary

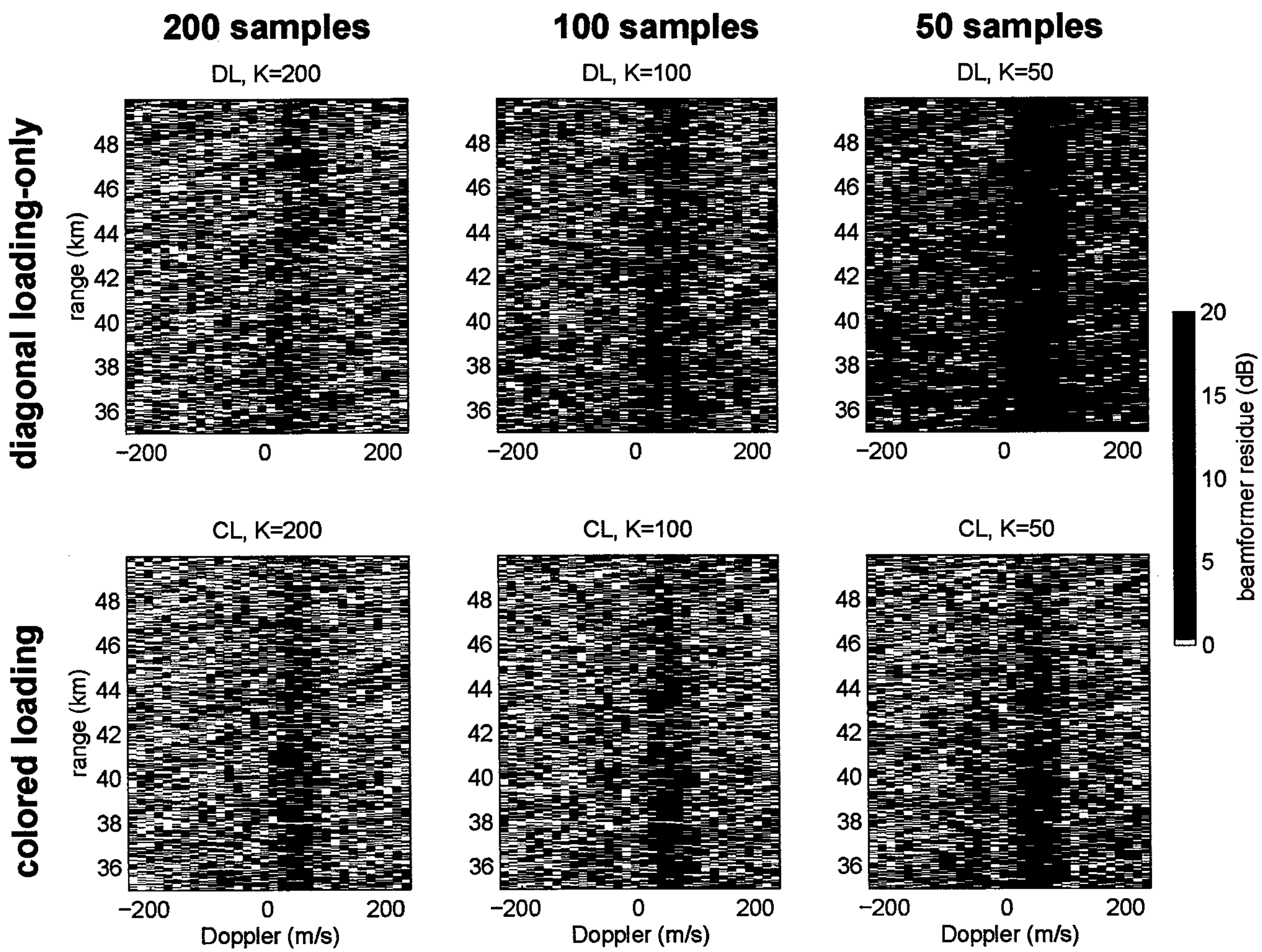


- **Same loading matrix assumed**
- **Doppler = 20 m/s (two-way)**
- **Improved convergence with colored loading**
- **The chosen Doppler bin is relatively close to the mainbeam clutter**
- **Range bin 584**

Technique works in the presence of calibration errors

[redacted]

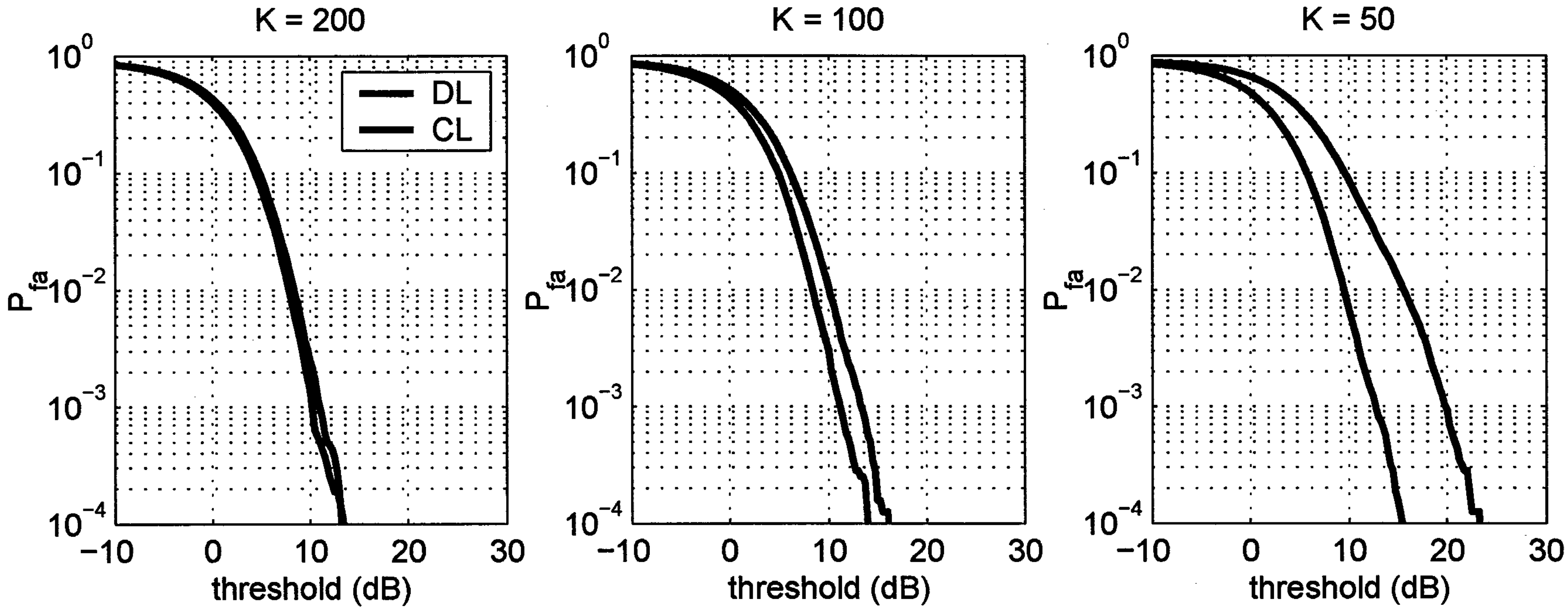
Beamformer Residue



- **Loading levels:**
  - $\beta_L = 0$ dB,
  - $\beta_d = 30$ dB
- **Colored loading beamformer is more robust to reductions in sample support**
- **Colored loading results in fewer false alarms**
- **Weights normalized s.t. $w^H w = 1$**
- **Thermal noise power is ~0 dB**

[redacted]

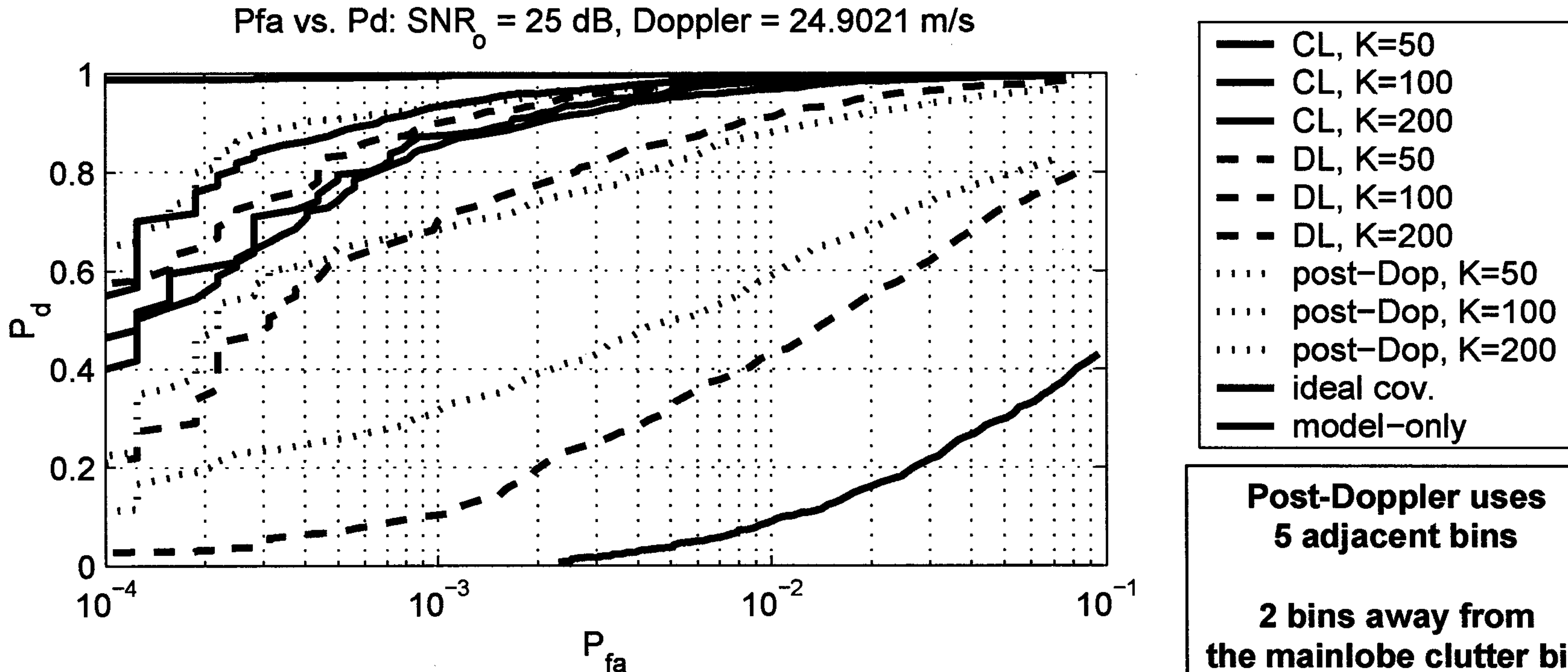
Raw False Alarm Summary



- **Fraction of pixels outside mainlobe clutter Doppler bins (3 bins) exceeding a given threshold is shown**
- **Colored loading beamformer maintains a similar false alarm characteristic as the number of training samples is decreased**

[redacted]

Detection Performance Summary
("endo-clutter")



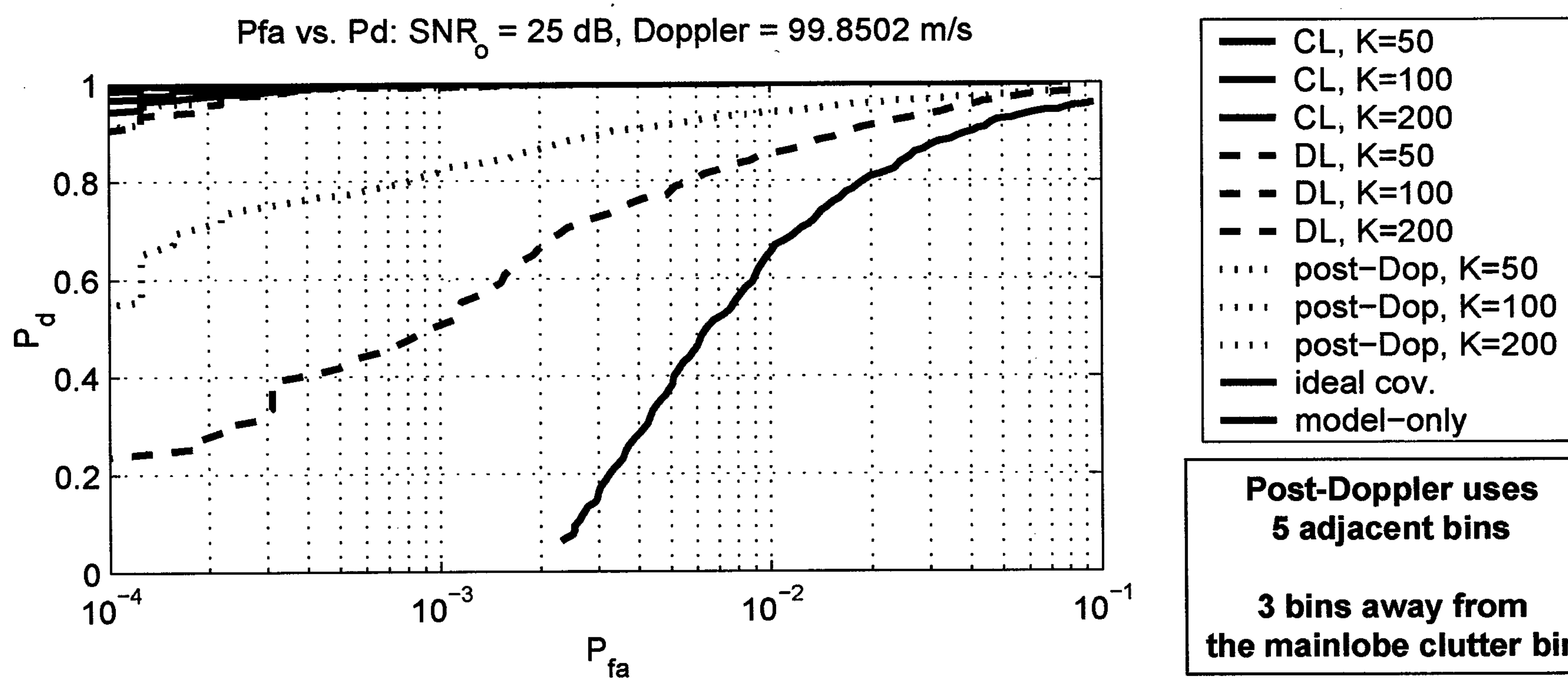
Post-Doppler uses 5 adjacent bins

2 bins away from the mainlobe clutter bin

- **Detector includes median CFAR normalization of the beamformer output prior to thresholding**
- **No targets in the secondary beamformer or CFAR training data**
- **1000 Injected test targets: all ranges, Doppler = 24.90 m/s, Target SNR is 25 dB at closest range bin (~5 dBsm)**
- **Colored loading beamformer is more robust as sample support is reduced**

[redacted]

Detection Performance Summary
("exo-clutter")



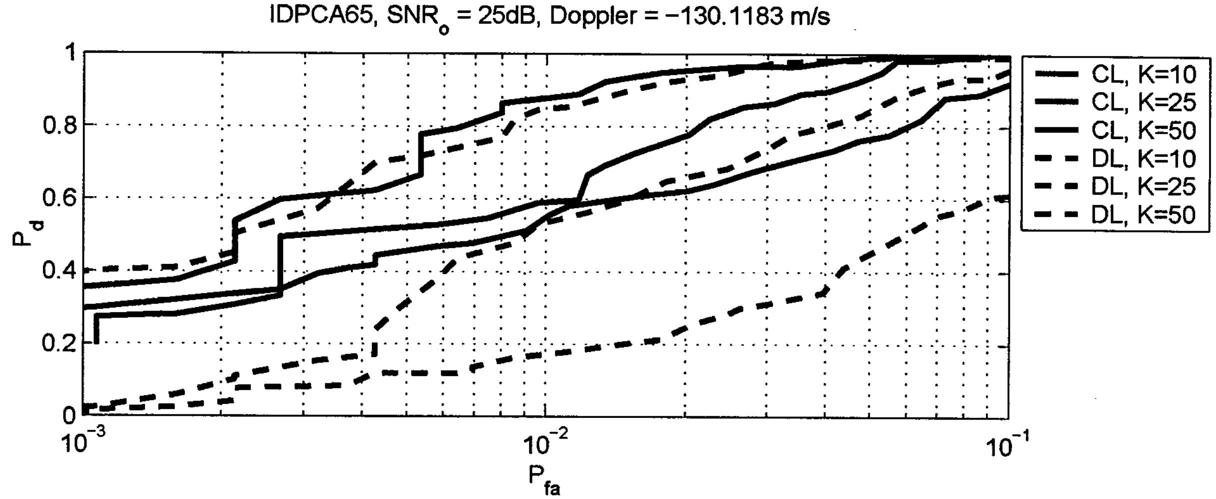
Post-Doppler uses 5 adjacent bins

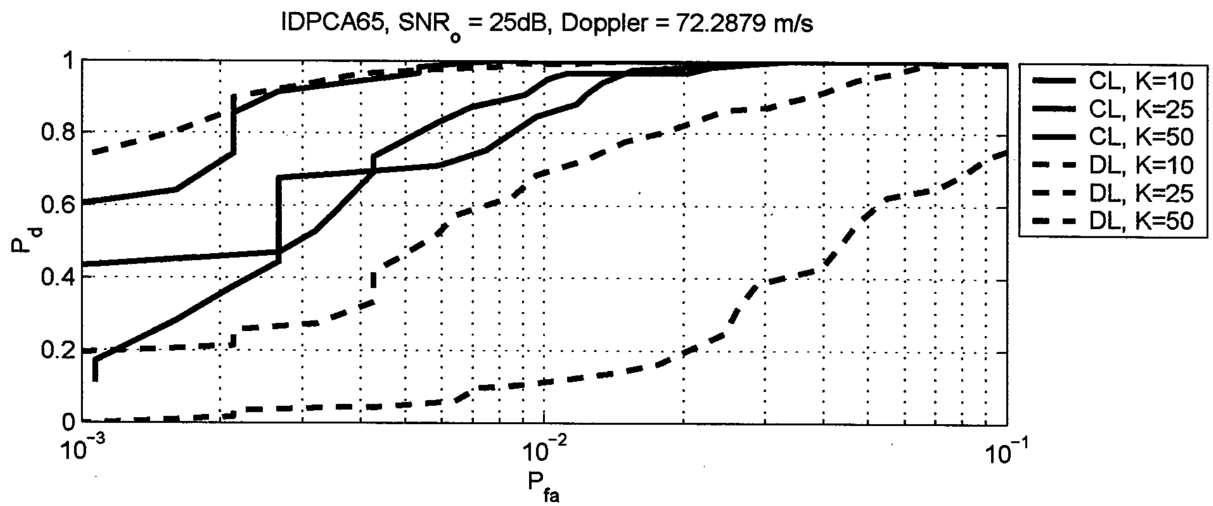
3 bins away from the mainlobe clutter bin

- **Same result as previous slide except injected target Doppler is 99.85 m/s**
- **Most of the beamformers with adaptivity perform well when target is separated from the mainbeam clutter**
- **Use the most computationally efficient algorithm in these Doppler bins**

[redacted]

Mountain Top IDPCA Experimental Data





Unknown system errors: calibration, multipath, transmitter instabilities

- **Data set parameters:**
  - UHF, PRF= 625 Hz
  - 178 range samples
  - 10 elems., 16 pulses
- **STAP: full-DoF SMI**
- **Boresight azimuth**
- **Two Doppler bins shown**
- **Same colored loading model used in previous results (β_L = 0dB, β_d = 40 dB)**
- **Similar trend as simulated data**
- **Injected test targets are 25 dB SNR at all ranges**

[redacted]

Summary

- A method for incorporating *a priori* knowledge in the space-time beamformer solution using linear or quadratic constraints has been presented
- Quadratic constraint solution results in "colored" loading which can be implemented efficiently in the data domain and offers a "blending" between adaptive and deterministic filtering
- The fidelity of the colored loading matrix will depend on the available *a priori* knowledge sources and computational resources
- The technique was applied to KASSPER site-specific simulation data and shown to result in more robust performance near the mainbeam clutter → improved MDV performance
- Similar performance trends observed with experimental data
- Extension to low-DoF STAP implementations (e.g., post-Doppler) is currently under way

[redacted]